Paper Summary: A Butterfly Subdivision Scheme for Surface Interpolation with Tension Control

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1 Method

In computer aided graphical design (CAGD), a desired shape for a curve or surface can be achieved by a set of control points $\{\vec{p}_i^0\}$ (vectors), together with a smoothing scheme. This paper generalises a four-point interpolatory sub-division scheme by Dyn et al.[1] to be applied to triangular polyhedral meshes. With the generalised scheme each recursive iteration doubles the number of control points (which quadrupels the number of triangles), and the refined triangulation retains the vertices of the coarser input. The calculation for each new control point is based on the vicinity of 8 previous points. The positioning of these points resembles the figure of a butterfly, so the method gets the name "butterfly scheme". In order to obtain a limit curve with C^1 smoothing ($k \to \infty$), results from Doo and Sabin [2], and Micchelli and Prautzch [3] restricts for each new control point q of the $(k+1)^{th}$ iteration.

$$\vec{q}^{k+1} = \frac{1}{2}(\vec{p}_1^k + \vec{p}_2^k) + \omega \vec{s} \tag{1}$$

Where $\vec{s} = 2(\vec{p}_3^k + \vec{p}_4^k) - (\vec{p}_4^k + \vec{p}_5^k + \vec{p}_6^k + \vec{p}_7^k)$.

$\vec{q}^{k+1}$ can be thought as the midpoint of $(\vec{p}_1^k, \vec{p}_2^k)$ "corrected" by $\omega\vec{s}$.

Local control is achieved by preassigning a tension value ω_i^0 to each control point $\vec{p}_i^0$ and by assigning recursively a tension value ω_i^k to a new point $\vec{p}_i^k$ by linear interpolation.

$$\omega = \frac{1}{2}(\omega_1^k + \omega_2^k) \tag{2}$$

ω is defined at each point as a tension matrix,

$$\omega = \begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix} \tag{3}$$

If local control is not desired ω may be set as a constant scalar for all iterations

2 Implementation

The implementation shows that a coarse selection of control points produce after four iterations smooth models that resemble a fish and a face-like object. Local tension is demonstrated on the fish-example to show that one can effect the result of the surface at localised points.

References

[1] Dyn N, Gregory J A, Levin D A: *A four-point interpolatory subdivision scheme for curve design*, Computer Aided Geometric Design 4 (1987), 257-268

[2] Doo D, Sabin M: *Behaviour of recursive division surface near extraordinary points*, Computer Aided Design 10 (1978), 356-360

[3] Micchelli C A, Prautzsch H: *Uniform refinement of curves*, Linear Algebra and Applications 114/115 (1989), 841-870